A Combined DGPS/INS and Synthetic Aperture Radar System for Geoid Referenced Elevation Models and Ortho-Rectified Image Maps

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ABSTRACT

The objective of this paper is to introduce and describe the concept of using airborne gravimetry by strapdown INS/DGPS to reference mapping products derived from Interferometric Synthetic Aperture Radar (IFSAR) to the geoid. Calgary-based Intermap Technologies Ltd. is a digital mapping company that focuses on providing digital elevation models and ortho-rectified images using an interferometric radar system called the STAR-3i. Intermap Technologies Ltd., in partnership with the University of Calgary, has developed a product capable of providing geoid referenced digital elevation models and ortho-rectified images.

Using the IFSAR technology, the STAR-3i provides a new generation of airborne radar digital elevation models and ortho-rectified image maps. The STAR-3i consists of two X-band radar antennas mounted on a LearJet 36. Data is collected simultaneously for the two antennae, and is combined by a digital correlation process to extract terrain information. STAR-3i uses integrated DGPS/INS navigation results to obtain highly accurate position and orientation control. Because DGPS is used as the position reference all terrain information is referred to the ellipsoid.

The University of Calgary has developed software that extracts gravity disturbance information from the DGPS/INS data collected by the STAR-3i system. Using the gravity information, a precise relative geoid is determined for the flight area. By combining this geoid with the EGM 96 geopotential model, the mapping products derived from the STAR-3i system can be directly referenced to the geoid.

The STAR-3i system, and the method of extracting the geoid from the integrated DGPS/INS data are presented. An analysis of geoid determination accuracy for system is given. The results are based on a flight test in California on August 23, 1997. Based on a preliminary analysis, for a flight elevation of 6000 metres and flight speed of 700 km/hr, the following results were obtained:

- 1.5 mGal RMS difference at 12 km half wavelength spatial resolution
- 2.0 mGal RMS difference at 9 km half wavelength spatial resolution.

These RMS values are calculated by comparison to upward continued ground gravity disturbances. The accuracy of the STAR-3i determined geoid is 5 centimetres (1σ) when compared with an independent geoid reference.
For the given flight conditions the accuracy of the STAR-3i elevation map is 0.5 to 1.5 metres (1σ).

1 INTRODUCTION

Height above sea level is the usual reference used for topographic height information. It indicates a physical definition of a height system. This reference surface is easy to visualize in ocean areas, but becomes more difficult to establish over land. The continuation of sea level below the land masses results in a surface called the geoid. The geoid is the proper reference surface for heights above sea level (or orthometric heights) and is therefore the basis of topographic maps. The geoid reference surface is physically meaningful and has the property that water flows downhill from the larger to the smaller height.

This is not necessarily true with other height systems. For example, heights determined by GPS refer to the surface of an ellipsoid which is a geometric rather than a physical surface. Although the ellipsoid approximates the geoid quite well on a global scale the deviations between them may be up to 100 m. Thus, in an ellipsoidal height system water may actually appear to flow uphill.

Since DGPS is used in the STAR-3i system for positioning, the resultant DEMs generated refer to ellipsoidal heights. Knowledge of the geoid in the flight area is therefore required to provide mapping products in an orthometric height system. Figure 1 gives a conceptual view of the difference between the ellipsoidal and orthometric height of a point.

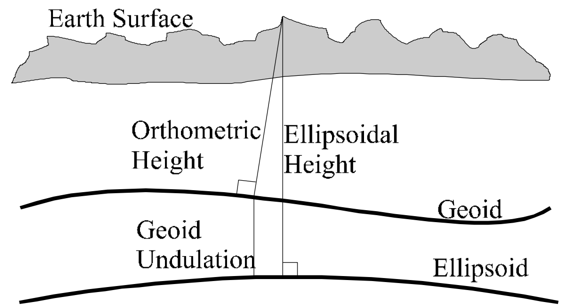

Figure 1: Height System Definitions

To transform the ellipsoidal DEMs obtained by STAR-3i to orthometric heights the difference between the ellipsoid and geoid must be computed. These differences are called geoid undulations. Normally, these are computed by combining satellite and terrestrial gravity observations. Geopotential models, such as EGM96, that combine satellite and terrestrial observations give a smooth global approximation of the geoid. EGM96 is a spherical harmonic expansion of the Earth's gravity field complete up to degree and order 360. However, the accuracy of this model for a certain area is dependent upon the amount of local gravity information from that area used in the calculation of the model. Therefore, for areas with poor terrestrial gravity coverage, the model would only give geoid undulations accurate to a couple of metres (see Figure 3). This accuracy is not sufficient for referring STAR-3i DEMs to orthometric heights.

Intermap Technologies Ltd., in partnership with the University of Calgary has developed a method to extract local gravity field information from the DGPS/INS data that is collected by the STAR-3i system. This local gravity field information is then used in combination with a global geopotential model to determine a precise geoid for the flight area. This allows the referencing of all mapping products directly to orthometric heights. The large advantage of this approach is that all data for geoid determination, thematic mapping and DEM's can be collected in a single pass.

2 STAR-3i SYSTEM DESCRIPTION

The STAR-3i system is composed of two X-band radar antennae that are mounted on a Learjet 36. Data is collected from the antennae simultaneously and stored using a VME based data collection system. The data rate for the two antennae is approximately 1 Gigabyte per minute. The terrain information is then extracted from the acquired radar data using a digital correlation process. The terrain height information is used to form a digital elevation model (DEM) and to ortho-rectify the radar image to remove all vertical height distortions. The height extraction process requires a very precise independent estimate of aircraft position, velocity and attitude at the time of data capture. The estimation of these navigational parameters is accomplished using a combined Differential Global Positioning System (DGPS)/ Strapdown Inertial Navigation System (INS). The DGPS portion of the navigation system consists of two Ashtech Z-XII GPS receivers. The GPS data is collected at a 2 Hz data rate. The INS utilized in the STAR-3i system is the Honeywell H-770 inertial system. The H-770 is a navigation-grade strapdown inertial system with a stand-alone performance of 0.8 nm/hr. The data collection rate of the H-770 is 1200 Hz. A picture of the Learjet 36 that has been modified to carry the STAR-3i system is given in Figure 2. Note the large dome on the underside of the aircraft. This structure houses the radar antennae and the H-770 inertial system.

Figure 2: Learjet 36 Modified to Carry STAR-3i System

The accuracy of the DEMs and Ortho-Rectified Image Maps provided by the STAR-3i system is variable and can be tailored to meet specific customer requirements. In a typical collection mode the system is flown at 6000 metres elevation with a velocity of 700 km/hr. The collection swath width at this altitude is 6 km. DEM acquisition accuracy with these flight characteristics is on the order of 0.5 to 1.5 metres (1σ).

3 DATA PROCESSING FOR GEOID DETERMINATION

3.1 Principle of Strapdown Airborne Gravimetry

The data from the strapdown INS and DGPS sub-systems can be combined to estimate gravity disturbances δg, which can then be used for geoid undulation determination. The gravity disturbances are an estimate of the difference between the gravity of the ellipsoid and that of the physical earth. These gravity disturbances can then be converted into geoid undulations (see section 3.3). The following discussion and equation closely follow that given in Glennie and Schwarz, 1998.

Newton's equation of motion in the gravitational field of the earth can be used to derive the principle for strapdown inertial scalar gravimetry (SISG). It is described by the following equation:

$$\delta g = f_u - \dot{v}_u + \left(\frac{v_e}{R_n + h} + 2\omega_e \cos\varphi\right) v_e + \frac{v_n^2}{R_m + h} - \gamma \qquad (1)$$

where f_u is the upward component of specific force (from the strapdown INS), v_e, v_n, v_u are the east, north and up components of the aircraft velocity (from DGPS), R_m and R_n are the meridian and prime vertical radii of curvature, φ and h are geodetic latitude and height (from DGPS), ω_e is the Earth rotation rate and γ is normal gravity. The sum of the third and fourth terms in equation (1) is often called the Eötvös correction.

3.2 DGPS/INS Data Processing

The processing of the DGPS/INS data for gravity disturbance estimation occurs in three steps. In the first step the DGPS/INS data are integrated using a decentralized Kalman filter approach in the software package STARNAV. The integration allows the estimation and correction of INS accelerometer biases and gyro drifts and the detection and repair of possible GPS cycle slips. In the second step the airborne gravity disturbances are calculated according to equation (1) in the software package STARGRAV. In order to reduce the effect of measurement noise the resulting gravity disturbance estimates are also lowpass filtered. For geoid determination a lowpass filter with cutoff frequency of either 0.011 Hz or 0.00833 Hz is used. These cutoff frequencies correspond to a 1/e response time of 90 and 120 seconds respectively.

The first two steps provide an estimate of the gravity disturbances along the flight trajectory. However, these estimates suffer from long term changes in the INS biases and drifts. Therefore, in order to reduce the effect of the long term INS errors a final step consisting of a crossover adjustment of the gravity disturbance estimates is added. Obviously the crossover adjustment requires flight lines in the cross track direction. A bias and drift for each flight line are calculated in the adjustment and therefore a minimum of two cross track flight lines are required. The differences at crossover points are formulated as (Glennie and Schwarz, 1997):

$$\Delta\delta g = s_i \Delta t_i + s_j \Delta t_j + b_i + b_j \qquad (2)$$

where $\Delta\delta g$ is the difference in gravity disturbance at the crossover point, s_i, s_j, b_i and b_j are the slopes and biases of lines i and j respectively, and Δt_i and Δt_j are the times along lines i and j from t = 0 to the crossover point. All crossover point differences are used to solve for a bias and slope of each flight line. The data is combined using a parametric least squares adjustment.

3.3 Geoid Determination from Gravity Disturbances

To determine the geoid from the estimated gravity disturbances the following procedure is employed:

1. The gravity disturbances computed along the flight trajectory are interpolated at grid points after the global geopotential model is removed, and the effect of the topography is removed using the STAR-3i generated DEM.
2. The gravity disturbances at flight level are downward continued to the geoid using the inverse of the Poisson integral.
3. The geoid undulations at ground level are computed from the downward continued gravity disturbances using Stokes' formula.
4. The effect of the topography and the global geopotential model is restored into the geoidal undulations.

This is only one of a number of possible methods that could be utilized to determine geoid undulations from gravity disturbances. Currently, a number of alternative approaches are also being investigated to determine the best method. For more information on downward continuation of airborne gravity data the interested reader is referred to Rubek, 1997 and Forsberg and Kenyon, 1995.

4 EXPECTED GEOID DETERMINATION ACCURACY

Equation (1) shows that the accuracy with which gravity disturbances can be determined is a function of INS specific force measurement and attitude errors and errors in GPS determined position, velocity and kinematic acceleration. Previous investigations (see for e.g. Brozena et al, 1989 or Schwarz et al, 1991) have shown that errors in DGPS position and velocity have a negligible effect on airborne gravity disturbance estimation. Therefore the errors in the GPS and INS accelerations can be analyzed to determine the expected geoid determination accuracy. A detailed analysis of this type is beyond the scope of this paper. However, such an analysis has been undertaken. The reader is referred to Schwarz and Li (1996) for the details of this analysis. A sketch of the results and theory is given below.

The power spectral density (PSD) of the airborne gravity disturbance error $S_{\delta g}(\lambda)$ can be related to the INS and GPS specific force and acceleration errors by the formula:

$$S_{\delta g}(\lambda) = S_{GPS}(\lambda) + S_{INS}(\lambda) \tag{3}$$

where S_{GPS} and S_{INS} are the PSD of the GPS acceleration erros and INS specific force errors respectively and λ is the wavelength. The PSD of the gravity disturbance errors can then be related to the PSD of the geoid undulation errors by the formula:

$$S_N(\lambda) = e^{4\pi h/\lambda}(2\pi\gamma)^{-2}\,\lambda^2 S_{\delta g}(\lambda) \tag{4}$$

where h is the ellipsoidal height and γ is normal gravity. The PSD of the geoid undulation errors can be easily converted into a cumulative geoid undulation error as a function of wavelength in kilometres. This calculation has been performed for the strapdown INS/DGPS configuration. The cumulative relative geoid undulation error of the system is shown in Figure 3 (labeled as STAR-3i Relative Geoid Accuracy). The accuracy of the geoid undulation predicted using the EGM96 geopotential model is also given in Figure 3 for areas with poor and very good terrestrial gravity coverage. Finally, the dotted lines indicate the derived absolute geoid accuracy when the relative geoid information of the STAR-3i system is combined with the EGM96 model. Note that in both cases the combination results in an improvement in the accuracy of the geoid. However, in the case of poor terrestrial gravity coverage, the improvement is much more significant. The best airborne gravity coverage in areas with good local gravity coverage is at about 250000 km^2 (500 km by 500 km), with a resulting absolute geoid accuracy at the 20 cm level. In areas with poor terrestrial gravity coverage the optimal combination area is about 640000 km^2 (800 km by 800 km) with a resulting absolute geoid accuracy of approximately 35 cm.

It is important to note that this is a theoretical analysis of geoid undulation model enhancements by the STAR-3i local gravity disturbance estimates. Although actual INS and DGPS data has been used to derive the power spectral densities of the geoid undulation errors, some assumptions about the EGM96 model have been made in the comparison. The actual level of improvement provided by the STAR-3i system would vary depending on location of flights and actual flight conditions. For example, the figure below was generated assuming no crossover lines for the flight. If periodic crossover lines were added then the STAR-3i relative geoid determination would be expected to be better than that depicted in Figure 3.

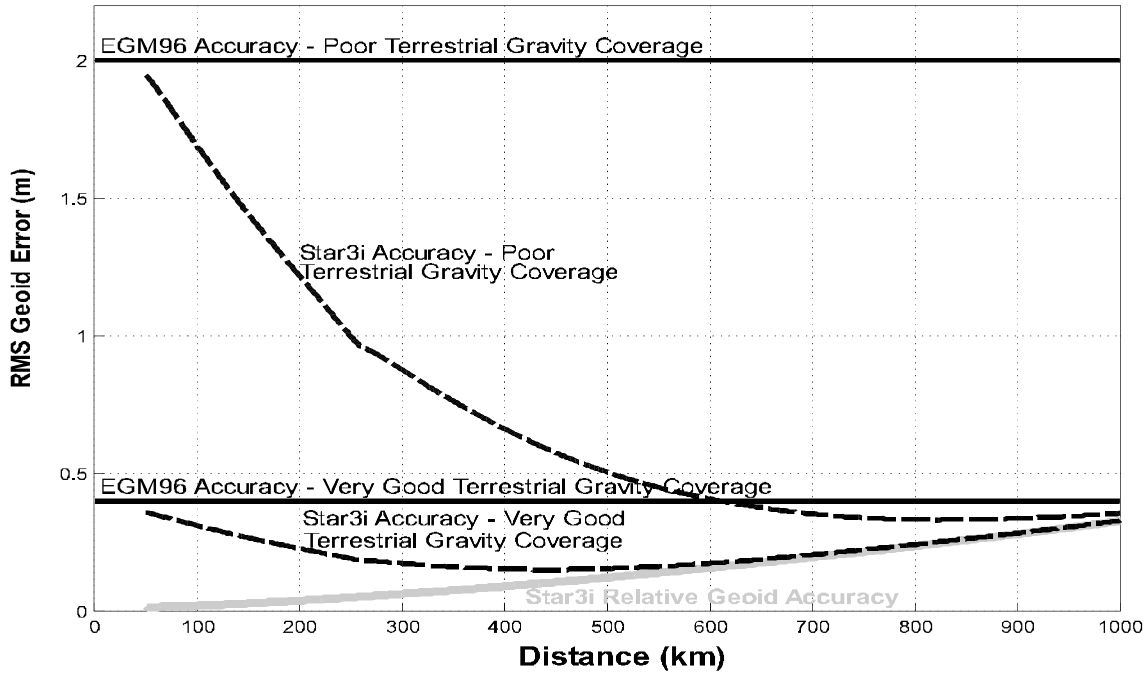

Figure 3: Theoretical Geoid Accuracy Derived from STAR-3i Data

5 RESULTS

5.1 Gravity Disturbance Estimation

The results presented in this section are from an airborne test of the Star-3i system near Sacramento California on August 23, 1997. Data for a total of twelve flight lines each of approximately 200 km length was collected and processed using the STARGRAV software. The aircraft was flown at a pressure altitude of approximately 6000 metres with a velocity of 700 km/h. More test details as well as a more detailed analysis of gravity disturbance estimation accuracy during the test can be found in Tennant et al, 1998. A summary of these results is presented below.

As an independent check of the gravity disturbance estimates produced by the STAR-3i DGPS/INS sub-system, reference gravity disturbances at flight elevation were computed from ground gravity measurements and an existing DEM of the flight area. The ground gravity measurements are terrain corrected free air anomalies on a 2' by 2' grid obtained from the NGS web site (www.ngs.noaa.gov). The DEM for the area was on a 5' by 5' grid. The data were combined using Poisson's integral to upward continue the gravity disturbances to flight level. More details on upward continuation calculations can be found in Argeseanu, 1995. The results of the comparison at flight level between the system estimates and the upward continued reference gravity disturbances is given in Table 1 (after Tennant et al, 1998). An average of results from the twelve flight lines is given for both the 90 and 120 second lowpass filters. Assuming a speed of 700 km/h this corresponds to a spatial resolution (half-wavelengh) of 9 km and 12 km respectively. Note that a linear bias between the reference and estimated gravity disturbances has been removed for all flight lines. This was necessary because this test did not have two flight lines in the cross track direction to allow the computation of a crossover adjustment to remedy long term INS errrors (see section 3.2). Additionally, Figure 4 shows a typical comparison between the estimate and the reference for one of the flight lines.

1/e Response	σ (mGal)
90 sec	1.5
120 sec	2.0

Table 1: Average Standard Deviation (s) of Twelve Flight Lines between STAR-3i Estimates and Upward Continued Gravity Disturbances (mGal)

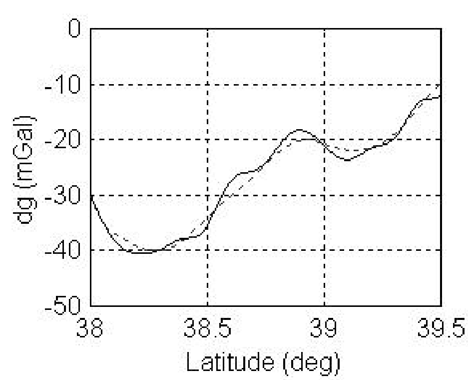

Figure 4: Comparison Between Upward Continued Reference (dotted) and STAR-3i Disturbance Estimate, Typical Flight Line

5.2 Geoid Undulation Determination

Using the California test data a relative geoid was determined for the flight area using the software package STARGEOID. This software package uses the computational procedures outlined in section 3.3. This relative geoid was compared to an existing precise geoid model for the flight area. The existing geoid model, GEOID96, was obtained from the NGS from their web site (www.ngs.noaa.gov). The accuracy of the GEOID96 model is given as 2.5 cm (1σ) for points spaced 50 km or greater (www.ngs.noaa.gov/GEOID/geoid96.html). The results of the comparison show that the STAR-3i geoid has an accuracy of 5 centimetres (1σ) when compared with the geoid reference. The maximum deviation was 9 centimetres (Wei et al, 1998). The geoid undulations in the flight area varied by approximately 8 metres, with a standard deviation of 2.7 metres.

6 CONCLUSIONS

In order to provide mapping products that are referenced to physically meaningful surface, such as the geoid, a model for geoid undulations to account for the differences between orthometric and ellipsoidal heights is required. Existing geopotential models such as EGM96 lack sufficient short wavelength information (especially in areas with poor terrestrial gravity coverage) to provide undulations that are accurate enough. Therefore, an additional source of local gravity field information is required. Intermap Technologies Ltd. and The University of Calgary have developed software to extract gravity disturbance information from the DGPS/INS data collected onboard the STAR-3i Synthetic Aperture Radar System. These gravity disturbances can be used to determine precise local geoid undulation information.

An analysis of flight data collected in August of 1997, the STAR-3i system recovers gravity disturbances with an accuracy of 1.5 mGal and 2.0 mGal at 12 and 9 km half wavelength spatial resolutions respectively when compared with a reference upward continued from ground gravity measurements. The resulting accuracy of the geoid model determined using these gravity disturbance estimates is 5 centimetres (1σ) when compared with an independent geoid reference.

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